Comparison of Measurements: A Bayesian Approach

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Introduction of the Question

The Tool of Choice

How the Heck is that Useful for our Question?

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Conclusions

The Question in a Nutshell

- Assessment of potential impacts of BC Hydro's Site C project near Fort St. John in British Columbia.
- Oct. 2004 – Sep. 2005 was selected to represent 'normal' climate from 1971-2000.
- Based on monthly data and primarily hydrological criteria.

Is this 1-year period sufficiently representative of normal climate to model local climate changes caused by filling the reservoir?

Defining “Representative of Normal Climate”

1. Mean should be statistically significantly the same as the climate average (unbiased).
2. Differences between 1-year period and climate average should be smaller than the 30-year climate variability: 1-year period closer to climate average than 1-year random samples drawn from 30-year climate normals.

Statistical Generalization

RWDI
CONSULTING ENGINEERS & SCIENTISTS

The basic statistical problem:

- Two data sets: samples from the same population?

Broad applicability:

- Measurements of contaminants up- and downstream of the oil sands region
- Reduction of ambient concentrations after implementation of emission controls
- Effectiveness of new drug: samples are control and treatment groups

The Usual Approach

Hypothesis testing with two samples:

- Differences between means
- Standard deviations known or estimated if unknown

Does not tell us…

…probability of different means; only yes/no based on confidence threshold (semantics)

…probability of different standard deviations (that **is** important!)

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Bayesian Two-Sample Comparison

Appendix C in Gregory (2005) provides solutions to 4 probabilities adding up to 1:

- *p(C,S)* same means, same stdev
- ***p(C,~S)* same means, different stdev**
- *p(~C,S)* different means, same stdev
- *p(~C,~S)* different means, different stdev

Easy? Here is the Catch!

$$p(C,\overline{S}|D_1,D_2,I) = \frac{K(2\pi)^{-N/2}\Gamma(N_1/2)\Gamma(N_2/2)}{16R_c[\log(R_\sigma)]^2}\int_L^H dc_1 U_1^{-\frac{N_1}{2}}\, U_2^{-\frac{N_2}{2}}$$

$$\times\left[Q\left(\frac{N_1}{2},\frac{U_1}{\sigma_H^2}\right)-Q\left(\frac{N_1}{2},\frac{U_1}{\sigma_L^2}\right)\right]$$

$$\times\left[Q\left(\frac{N_2}{2},\frac{U_2}{\sigma_H^2}\right)-Q\left(\frac{N_2}{2},\frac{U_2}{\sigma_L^2}\right)\right].$$

where

$$U_1 = \frac{N_1}{2}(\overline{d_1^2} - 2c_1\overline{d_1} + c_1^2), \qquad U_2 = \frac{N_2}{2}(\overline{d_2^2} - 2c_1\overline{d_2} + c_1^2)$$

Additional Work Required

- Gregory (2005) provides Mathematica code
- Normalize at the end
- There are robust priors after standardization
- Drop terms common across all 4 probabilities
- Group by common exponents
- Replace gamma function by Stirling approximation
- Apply logarithm
- Use built-in function (incomplete gamma function) and integrations (Simpson quadrature)
- Take exponential just before normalization

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Additional Constraints

- Need 12 consecutive months
- Order matters (seasonal cycle):
  - Means differ
  - Standard deviations differ
- Six climate parameters, but only four are independent:
  - Min, max, mean temperature
  - Total snow, rain, precipitation

Data Standardization

Standardize:

1. For each calendar month, calculate 1971-2000 climate average and standard deviation.
2. For each month in Oct. 2004 – Sep. 2005, subtract the month's climate average and divide by the month's climate standard deviation.

After Standardization

1. The mean should be 0 with high probability.
2. The standard deviation should be <1 with high probability.

Implementation of Two-Sample Comparison

After standardization, compare:

1. 12 monthly values of the 1-year period
2. 360 values randomly drawn from standard normal distribution (30 years with 12 calendar months)
3. Repeat 10,000 times and calculate average probabilities

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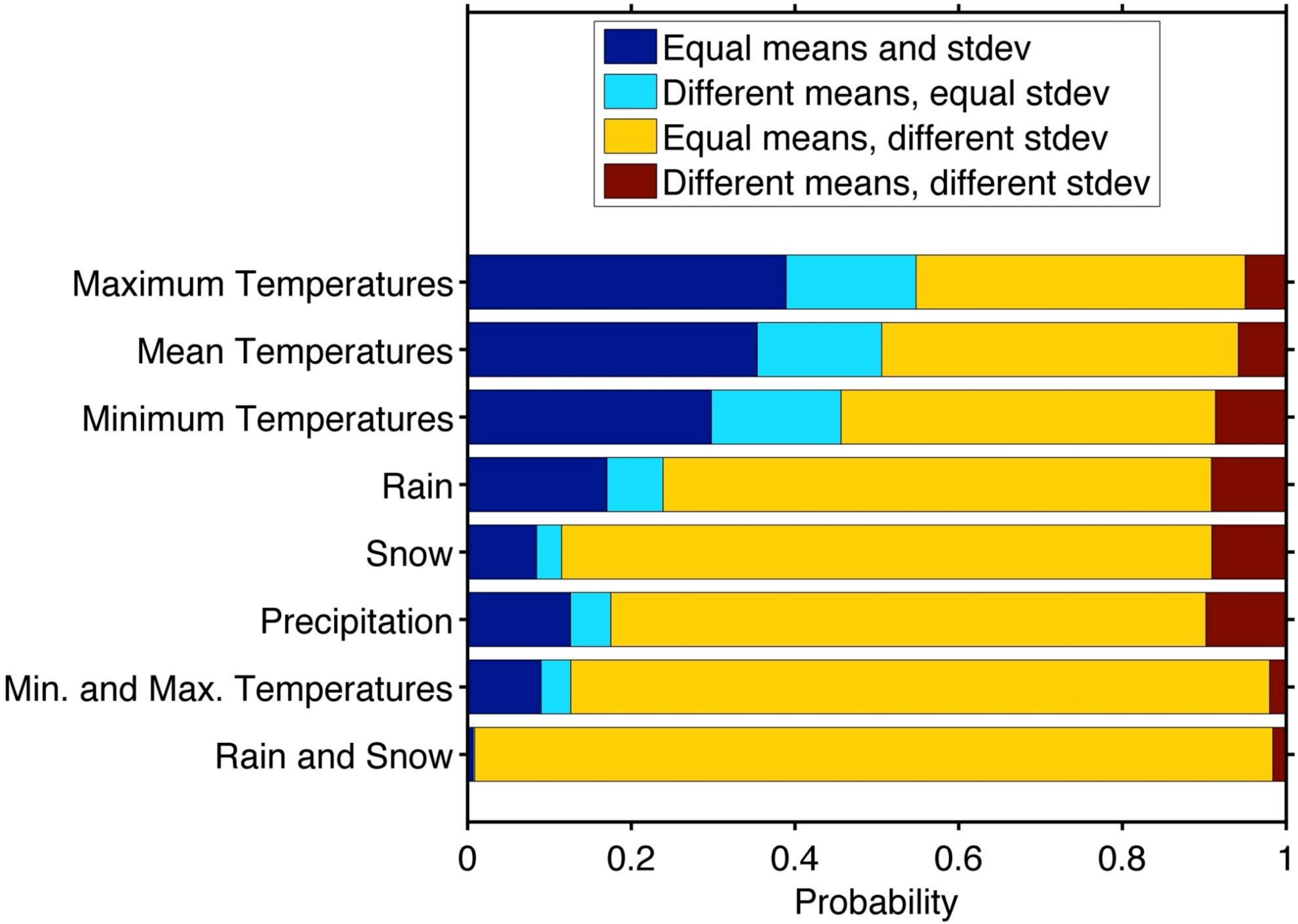

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- Bayesian approach:
  - continuum of probabilities rather than threshold
  - offers insights into 'significance' of variability
  - mathematically challenging
  - broad applicability
- Can treat individual or combined parameters
- Study year representative of climate normals for
  - rain, snow, and all precipitation
  - joined minimum and maximum temperature
- Study year okay for individual temperatures

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References and Further Reading

- Gregory, Phil, 2005: Bayesian Logical Data Analysis for the Physical Sciences. A Comparative Approach with Mathematica Support. Cambridge University Press, United Kingdom. 468 pages.